Photonics Center, Laboratory for Photoacoustics,
Institute of Physics, Belgrade, Serbia

L1 – Transport Processes in Physics

Dragan D. Markushev

Laboratory for Photoacoustics
Photonics Center
Institute of Physics, University of Belgrade,
Pregrevica 118,
11080 Belgrade-Zemun,
SERBIA

As a result of a non-equilibrium state, any physical system tends to return to equilibrium.

This return process is followed by some kind of transport of particles, heat, charges, etc.

Transport processes are usually called rate processes due to their time-dependence nature.

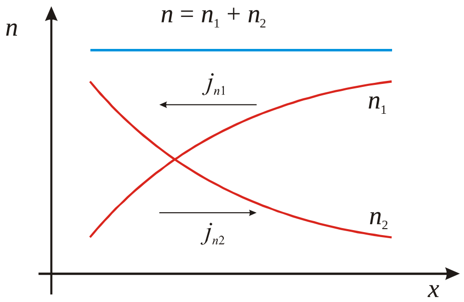
In physics, we are dealing with mass and energy transport in terms of particle and heat transfer, respectively.

Particle transfer is usually explained in terms of diffusion and heat transfer is usually explained in terms of thermal conductivity.

Constitutive equations (fluxes) for both of them are first established empirically:

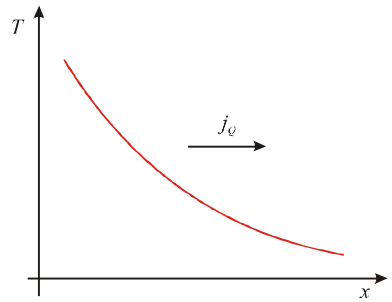
diffusion

thermal conductivity



$$j_{n_i} = -D\frac{\partial n_i}{\partial x}$$

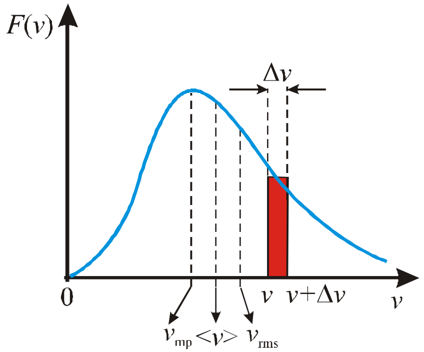
$$j_Q = -k\frac{\partial T}{\partial x}$$



Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

$$\Delta N = F(v)\Delta v$$



In physics, Maxwell distribution *F(v)* is a particular probability distribution first defined and used for describing particle speeds *v* in ideal gases.

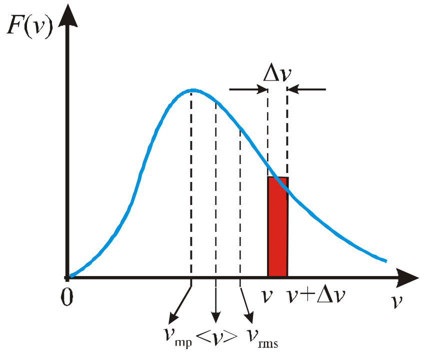
The term "particle" in this context refers to gaseous particles (atoms and molecules) and the system of particles is assumed to have reached thermodynamic equilibrium.

PHOTONICS CENTER

Laboratory for Photoacoustics
Photonics Center

Transport Processes
Molecular-Kinetic Explanation

$$\Delta N = F(v)\Delta v$$



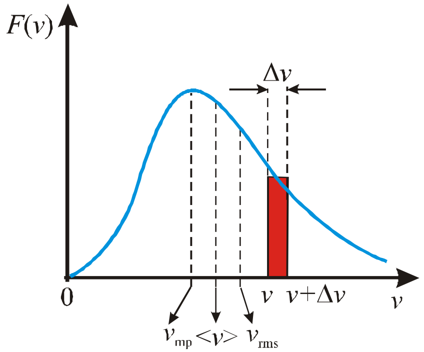
In thermodynamic equilibrium there are no net macroscopic flows of mass or energy, either within a system or between systems; macroscopic equilibrium exists.

In a macroscopic equilibrium, almost or perfectly exactly balanced microscopic exchanges occur; this is the physical explanation of the notion of macroscopic equilibrium.

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

$$\Delta N = F(v)\Delta v$$



A system in thermodynamic equilibrium has a spatially uniform intensive properties (temperature, pressure, density,…). All of them may be driven to local spatial inhomogeneity (non-equilibrium).

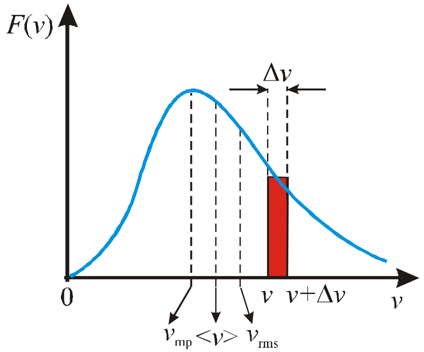
In non-equilibrium there are net flows of mass or energy, either within a system or between systems; transport processes occurs.

Laboratory for Photoacoustics
Photonics Center

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

$$\Delta N = F(v)\Delta v$$



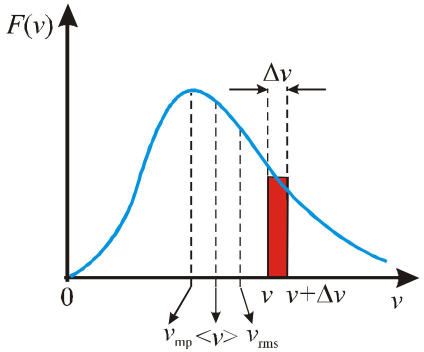
The Maxwell distribution is a result of the kinetic theory of gases which provides a simplified explanation of many fundamental gaseous properties in the terms of particle movement (*pressure* and *diffusion*).

Diffusion is particle transfer process based on the net movement of particles as a result of their random motion. Diffusion also can explain heat transfer as well.

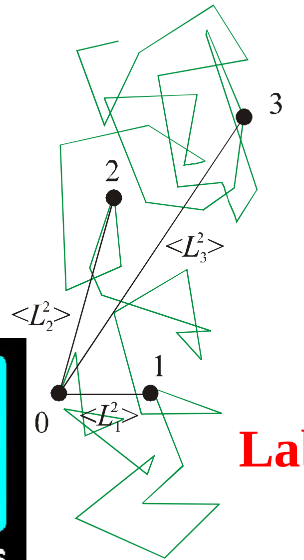
Laboratory for Photoacoustics
Photonics Center

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation



By particle “random motion” we mean that any molecule can move in any direction with equal probability.



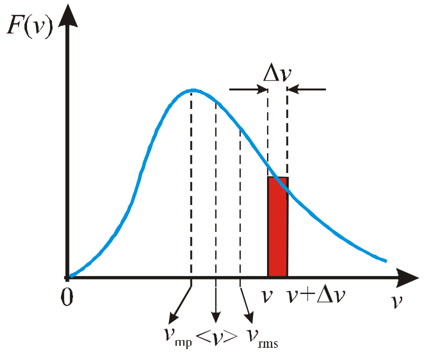
Particle “migration” represents the essence of the diffusion process.

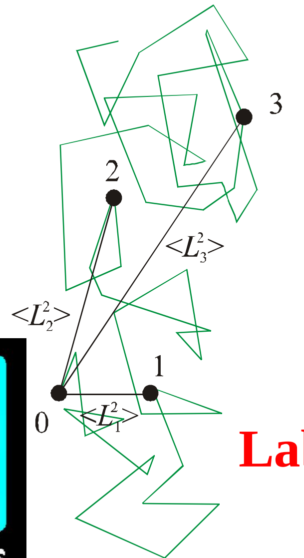
$$\langle L^2 \rangle \sim t$$

Laboratory for Photoacoustics
Photonics Center

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation





Einstein found that the mean-square displacement can be expressed in terms of the observation time, properties of the gas and the size of the particles.

$$\langle L^2 \rangle \sim t \rightarrow \langle L^2 \rangle = Dt$$

$\langle L^2 \rangle$ – mean-square displacement
t – observation time
D – diffusion coefficient

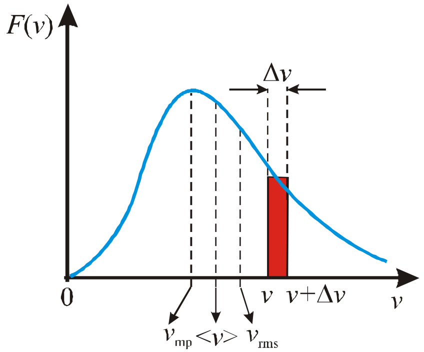
Laboratory for Photoacoustics
Photonics Center

Transport Processes
Molecular-Kinetic Explanation

Particle migration is a kind of transport process in which particle moves from one point in a system to another carrying, in the same time, its physical properties, too. For this kind of transport it is customary to define a particle flux by determining the rate at which the corresponding physical quantity of particle appears to move across an imaginary unit surface in the system because of the decrease in the conserved quantity on one side of the surface and the corresponding increase on the other.

Transport Processes
Molecular-Kinetic Explanation

In a gas, particle flux is easy to specify because physical properties carried by gas molecules crossing an imaginary unit surface in a unit time may be readily determined in terms of the velocities and concentrations of the molecules.

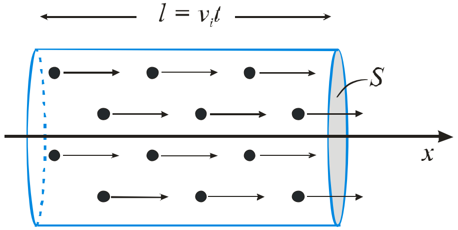


$$\Delta N_i = F(v)\Delta v_i \underset{\Delta v_i \to v_i}{\longrightarrow} N_i = F(v) v_i$$

$$\Delta n_i \to n_i, \ \sum_i n_i = n,$$

Transport Processes
Molecular-Kinetic Explanation

In a gas, particle flux is easy to specify because physical properties carried by gas molecules crossing an imaginary unit surface in a unit time may be readily determined in terms of the velocities and concentrations of the molecules.



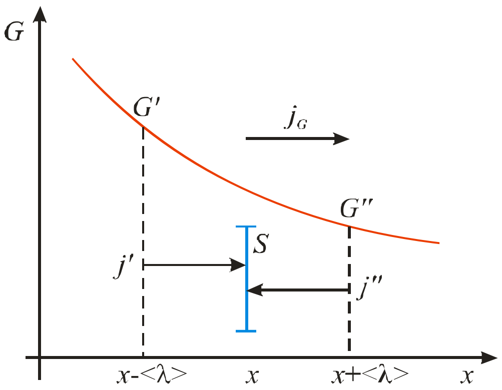
$$N_i = \frac{1}{6} n_i V = \frac{1}{6} n_i S l = \frac{1}{6} n_i S v_i t \qquad j_i = \frac{N_i}{St} = \frac{1}{6} n_i v_i ,$$

$$j = \sum_i j_i = \frac{1}{6} \sum_i n_i v_i \cdot \frac{n}{n} = \frac{1}{6} \frac{\sum_i n_i v_i}{n} \cdot n$$

$$j = \frac{1}{6} \langle v \rangle n$$

Transport Processes
Molecular-Kinetic Explanation

Changes of physical properties *G* carried by gas molecules crossing an imaginary unit surface *S* in a unit time are determined by general transport equation j_G:

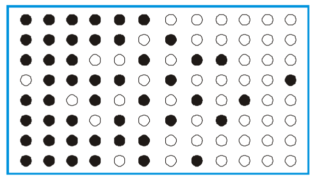


$$\left(j' = j'' = j\right)$$

$$j_G = j'G' - j''G'' = j\left(G' - G''\right) = \frac{1}{6}\langle v\rangle n\left(G' - G''\right).$$

$$G' - G'' = -\frac{\partial G}{\partial x} 2\langle \lambda \rangle,$$

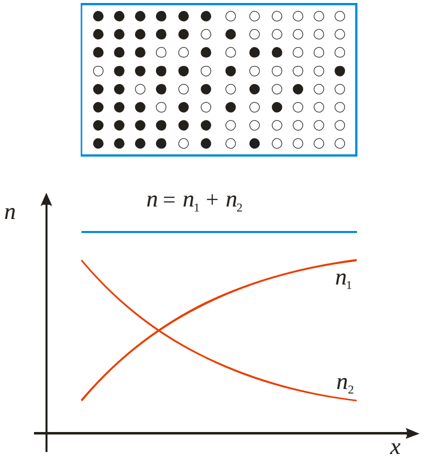
$$j_G = -\frac{1}{3} n \langle v\rangle \langle \lambda \rangle \frac{\partial G}{\partial x}.$$



Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

Transport processes in gases are often of great practical importance. The concentration changes of fuel in air will affect the performances of gasoline engine.



$$G = n_1 / n,$$

$$j_G = -\frac{1}{3} n \langle v \rangle \langle \lambda \rangle \frac{\partial G}{\partial x} = -\frac{1}{3} n \langle v \rangle \langle \lambda \rangle \frac{\partial \left(\frac{n_1}{n} \right)}{\partial x}.$$

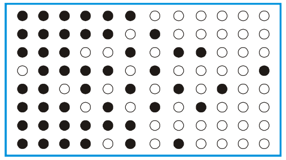
$$j_{n_1} = -\frac{1}{3} \langle v \rangle \langle \lambda \rangle \frac{\partial n_1}{\partial x}.$$

Laboratory for Photoacoustics
Photonics Center

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

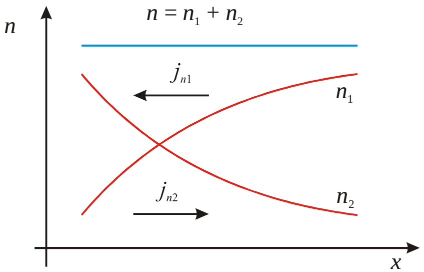
Transport Processes
Molecular-Kinetic Explanation

Transport processes in gases are often of great practical importance. The concentration changes of fuel in air will affect the performances of gasoline engine.



$$j_{n_1} = -\frac{1}{3}\langle v\rangle\langle\lambda\rangle\frac{\partial n_1}{\partial x} = -D\frac{\partial n_1}{\partial x}.$$

$$D = \frac{1}{3}\langle v\rangle\langle\lambda\rangle$$



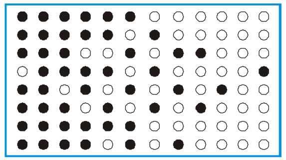
$$j_n = -D\frac{\partial n}{\partial x}$$

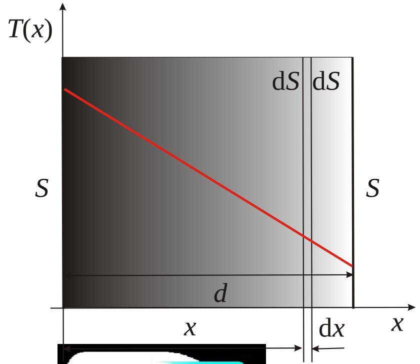
Fick's first law of diffusion

Laboratory for Photoacoustics
Photonics Center

Transport Processes
Molecular-Kinetic Explanation

Heat conduction in a gas will determine the rate of heating of a space vehicle entering the atmosphere of the Earth as well as the amount of isolator surface required for cooling.





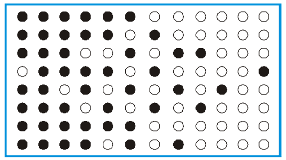
$$G = Q = \frac{3}{2}kT,$$

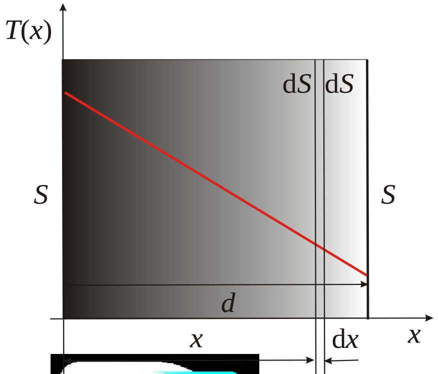
$$j_G = -\frac{1}{3}n\langle v\rangle\langle\lambda\rangle\frac{\partial G}{\partial x} = -\frac{1}{3}n\langle v\rangle\langle\lambda\rangle\frac{\partial\left(\frac{3}{2}kT\right)}{\partial x}.$$

$$j_{n_1} = -\frac{1}{3}n\langle v\rangle\langle\lambda\rangle\frac{3}{2}k\frac{\partial T}{\partial x}.$$

Transport Processes
Molecular-Kinetic Explanation

In an undeformed solid, the internal energy of the solid is changed only by the addition of heat and where no work is done, the internal energy flux is simply called heat flux.





$$j_{n_1} = -\frac{1}{3} n \langle v \rangle \langle \lambda \rangle \frac{3}{2} k \frac{\partial T}{\partial x}, \quad n = N_A / V, \quad \frac{3}{2} k \frac{N_A}{V} = c_V \rho$$

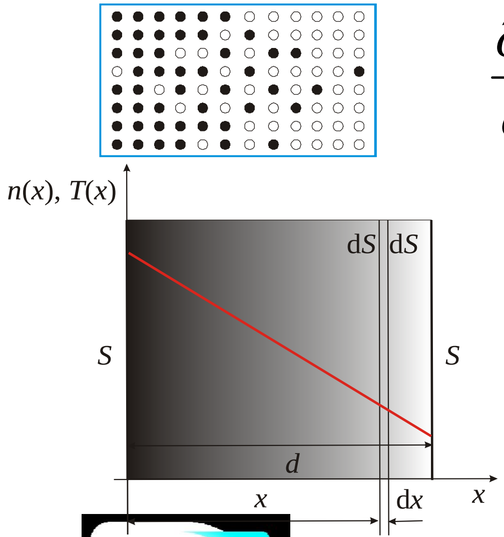
$$j_Q = -\frac{1}{3} \langle v \rangle \langle \lambda \rangle \rho c_V \frac{\partial T}{\partial x}, \qquad \boxed{k = \frac{1}{3} \langle v \rangle \langle \lambda \rangle \rho c_V .}$$

$$\boxed{j_Q = -k \frac{\partial T}{\partial x}}$$ Fourier's law of heat conduction

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

How diffusion causes the concentration and temperature to change with time?



$$\frac{\partial n}{\partial t} = \frac{1}{\partial x}\left[j_n(x) - j_n(x + \mathrm{d}x)\right] = -\frac{\partial j_n}{\partial x},$$

$$j_n = -D\frac{\partial n}{\partial x},$$

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

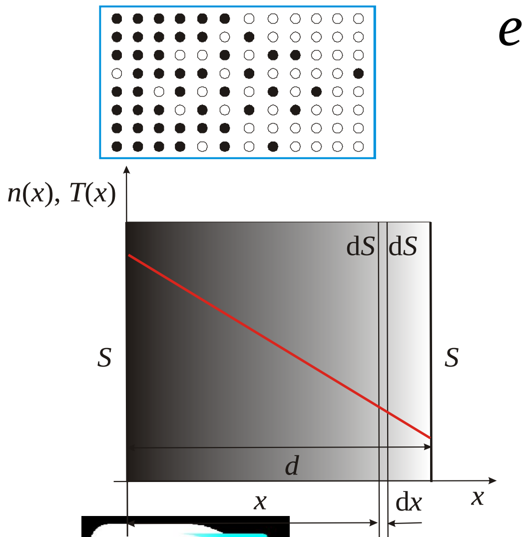
Fick's second law of diffusion

Laboratory for Photoacoustics
Photonics Center

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

How diffusion causes the concentration and temperature to change with time?



$$e = U/V = \rho c_V T, \qquad \frac{\partial e}{\partial t} = -\frac{\partial j_Q}{\partial x}, \qquad j_Q = -k\frac{\partial T}{\partial x},$$

$$\rho c_V \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2}, \qquad D_T = \frac{k}{\rho c_V}$$

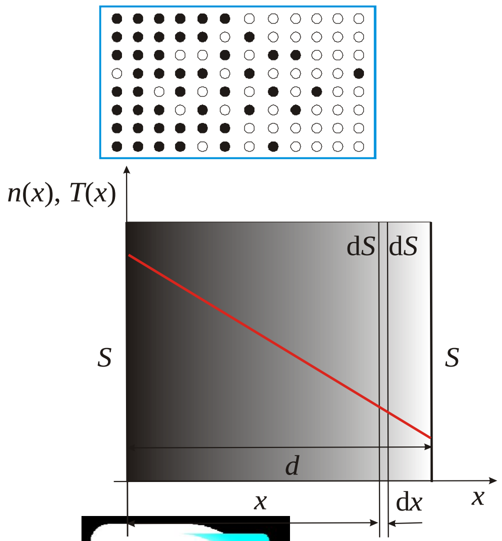
$$\frac{\partial T}{\partial t} = D_T \frac{\partial^2 T}{\partial x^2}$$

Heat equation

Laboratory for Photoacoustics
Photonics Center

Transport Processes
Molecular-Kinetic Explanation

Fundamental transport equations that describes the distribution of particles and heat in a given region over time



$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

Fick's second law of diffusion

$$[D] = [D_T] = [\langle v \rangle][\langle \lambda \rangle] = \frac{\mathrm{m}^2}{\mathrm{s}}$$

$$\frac{\partial T}{\partial t} = D_T\frac{\partial^2 T}{\partial x^2}$$

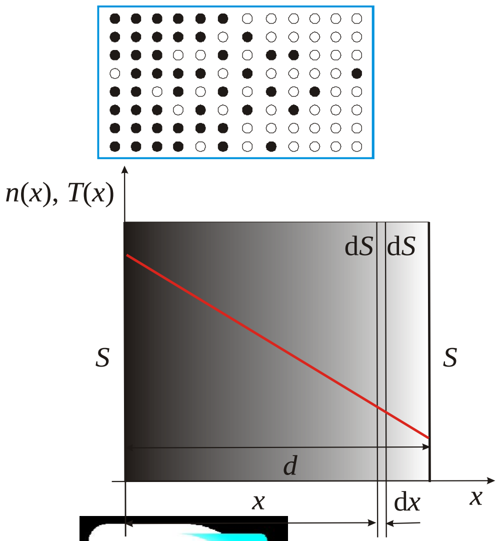
Heat equation

Laboratory for Photoacoustics
Photonics Center

Institute of Physics Belgrade,
Pregrevica 118, 11080 Belgrade-Zemun, SERBIA

Transport Processes
Molecular-Kinetic Explanation

Fundamental transport equations in the case of surface and volume absorbers (light/matter interaction).



Fick's second law of diffusion

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

$$\frac{\partial n}{\partial t} - D\frac{\partial^2 n}{\partial x^2} = \frac{\beta I}{\varepsilon}e^{-\beta x} - \frac{n}{\tau}$$

Heat equation

$$\frac{\partial T}{\partial t} = D_T\frac{\partial^2 T}{\partial x^2}$$

$$D_T\frac{\partial^2 T}{\partial x^2} - \frac{\partial T}{\partial t} = -\frac{H}{\rho c}$$

Laboratory for Photoacoustics
Photonics Center